

- 1 a** $(4 + i)^2 = 16 + 8i + i^2$
 $= 15 + 8i$
- b** $(2 - 2i)^2 = 4 - 8i + 4i^2$
 $= -8i$
- c** $(3 + 2i)(2 + 4i) = 6 + 12i + 4i + 8i^2$
 $= -2 + 16i$
- d** $(-1 - i)^2 = 1 + 2i + i^2$
 $= 2i$
- e** $(\sqrt{2} - \sqrt{3}i)(\sqrt{2} + \sqrt{3}i) = 2 - 3i^2$
 $= 2 + 3$
 $= 5$
- f** $(5 - 2i)(-2 + 3i) = -10 + 15i + 4i - 6i^2$
 $= -4 + 19i$

- 2 a** $z = 2 - 5i$
 $\bar{z} = 2 + 5i$
- b** $z = -1 + 3i$
 $\bar{z} = -1 - 3i$
- c** $z = \sqrt{5} - 2i$
 $\bar{z} = \sqrt{5} + 2i$
- d** $z = 0 - 5i$
 $\bar{z} = 0 + 5i = 5i$

- 3 a** $\bar{z}_1 = 2 + i$
- b** $\bar{z}_2 = -3 - 2i$
- c** $z_1 z_2 = (2 - i)(-3 + 2i)$
 $= -6 + 4i + 3i - 2i^2$
 $= -4 + 7i$
- d** $\overline{z_1 z_2} = -4 - 7i$
- e** $\bar{z}_1 \bar{z}_2 = (2 + i)(-3 - 2i)$
 $= -6 - 4i - 3i - 2i^2$
 $= -4 - 7i$
- f** $z_1 + z_2 = (2 - i) + (-3 + 2i)$
 $= -1 + i$
- g** $\overline{z_1 + z_2} = -1 - i$
- h** $\bar{z}_1 + \bar{z}_2 = (2 + i) + (-3 - 2i)$
 $= -1 - i$

4 $z = 2 - 4i$

a $\bar{z} = 2 + 4i$

$$\begin{aligned}\text{b} \quad z\bar{z} &= (2 - 4i)(2 + 4i) \\ &= 4 - 16i^2 \\ &= 20\end{aligned}$$

$$\begin{aligned}\text{c} \quad z + \bar{z} &= (2 - 4i) + (2 + 4i) \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{d} \quad z(z + \bar{z}) &= 4z \\ &= 8 - 16i\end{aligned}$$

$$\begin{aligned}\text{e} \quad z - \bar{z} &= (2 - 4i) - (2 + 4i) \\ &= -8i\end{aligned}$$

$$\begin{aligned}\text{f} \quad i(z - \bar{z}) &= i \times -8i \\ &= -8i^2 = 8\end{aligned}$$

$$\begin{aligned}\text{g} \quad z^{-1} &= \frac{1}{2 - 4i} \\ &= \frac{1}{2 - 4i} \times \frac{2 + 4i}{2 + 4i} \\ &= \frac{2 + 4i}{4 - 16i^2} \\ &= \frac{2 + 4i}{20} \\ &= \frac{1}{10}(1 + 2i)\end{aligned}$$

$$\begin{aligned}\text{h} \quad \frac{z}{i} &= \frac{z}{i} \times \frac{i}{i} \\ &= \frac{i(2 - 4i)}{-1} \\ &= -1 \times (2i - 4i^2) \\ &= -4 - 2i\end{aligned}$$

$$\begin{aligned}5 \quad (a + bi)(2 + 5i) &= 2a + 5ai + 2bi - 5b \\ &= 3 - i\end{aligned}$$

$$2a - 5b = 3$$

$$5a + 2b = -1$$

Multiply the first equation by 2 and the second equation by 5.

$$4a - 10b = 6 \quad \textcircled{1}$$

$$25a + 10b = -5 \quad \textcircled{2}$$

$\textcircled{1} + \textcircled{2}$:

$$29a = 1$$

$$a = \frac{1}{29}$$

$$\frac{2}{29} - 5b = 3$$

$$5b = \frac{2}{29} - 3$$

$$= -\frac{85}{29}$$

$$b = -\frac{17}{29}$$

$$\begin{aligned}
 \text{6 a } \frac{2-i}{4+1} &= \frac{2-i}{4+1} \times \frac{4-i}{4-i} \\
 &= \frac{8-2i-4i+i^2}{16-i^2} \\
 &= \frac{7-6i}{17} \\
 &= \frac{7}{17} - \frac{6}{17}i
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \frac{3+2i}{2-3i} &= \frac{3+2i}{2-3i} \times \frac{2+3i}{2+3i} \\
 &= \frac{6+9i+4i+6i^2}{4-9i^2} \\
 &= \frac{13i}{13} = i
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \frac{4+3i}{1+i} &= \frac{4+3i}{1+i} \times \frac{1-i}{1-i} \\
 &= \frac{4-4i+3i-3i^2}{1-i^2} \\
 &= \frac{7-i}{2} \\
 &= \frac{7}{2} - \frac{1}{2}i
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \frac{2-2i}{4i} &= \frac{2-2i}{4i} \times \frac{i}{i} \\
 &= \frac{2i-2i^2}{-4} \\
 &= \frac{2+2i}{-4} \\
 &= \frac{-1-i}{2} \\
 &= -\frac{1}{2} - \frac{1}{2}i
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \frac{1}{2-3i} &= \frac{1}{2-3i} \times \frac{2+3i}{2+3i} \\
 &= \frac{2+3i}{4-9i^2} \\
 &= \frac{2+3i}{13} \\
 &= \frac{2}{13} + \frac{3}{13}i
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \frac{i}{2+6i} &= \frac{i}{2+6i} \times \frac{2-6i}{2-6i} \\
 &= \frac{2i+6}{4-36i^2} \\
 &= \frac{2i+6}{40} \\
 &= \frac{3}{20} + \frac{1}{20}i
 \end{aligned}$$

$$(3-i)(a+bi) = 3a + 3bi - ai + b$$

$$= 6 - 7i$$

$$3a + b = 6 \quad \textcircled{1}$$

$$-a + 3b = -7$$

$$-3a + 9b = -21 \quad \textcircled{2}$$

$\textcircled{1} + \textcircled{2}$:

$$10b = -15$$

$$b = -\frac{3}{2}$$

$$3a - \frac{3}{2} = 6$$

$$3a = 6 + \frac{3}{2} = \frac{15}{2}$$

$$a = \frac{5}{2}$$

8

a

$$z = \frac{42i}{2-i}$$

$$= \frac{42i}{2+i} \times \frac{2+i}{2+i}$$

$$= \frac{84i + 42i^2}{4-i^2}$$

$$= \frac{-42 + 84i}{5}$$

$$= -\frac{42}{5} + \frac{84i}{5}$$

b

$$z = \frac{-2-i}{1+3i}$$

$$= \frac{-2-i}{1+3i} \times \frac{1-3i}{1-3i}$$

$$= \frac{-2+6i-i+3i^2}{1-9i^2}$$

$$= \frac{-5+5i}{10}$$

$$= -\frac{1}{2}(1-i)$$

c

$$z = \frac{1+i}{5+3i}$$

$$= \frac{1+i}{5+3i} \times \frac{5-3i}{5-3i}$$

$$= \frac{5-3i+5i-3i^2}{25-9i^2}$$

$$= \frac{8+2i}{34}$$

$$= \frac{1}{17}(4+i)$$

d
$$\begin{aligned} z &= \frac{5 + 2i}{2(4 - 7i)} \\ &= \frac{5 + 2i}{2(4 - 7i)} \times \frac{4 + 7i}{4 + 7i} \\ &= \frac{20 + 35i + 8i + 14i^2}{2(16 - 49i^2)} \\ &= \frac{6 + 43i}{130} \\ &= \frac{1}{130}(6 + 43i) \end{aligned}$$

e
$$\begin{aligned} z &= \frac{4}{1 + i} \\ &= \frac{4}{1 + i} \times \frac{1 - i}{1 - i} \\ &= \frac{4 - 4i}{1 - i^2} \\ &= \frac{4 - 4i}{2} \\ &= 2 - 2i \end{aligned}$$